Universal Journal of Applied Mathematics 4(1) 22-31: 2016 http://www.hrpub.org

DOI:10.13189/Ujam.2016.040103

Sum of Three Biquadatics a Multiple of a n^{th} Power, n = (2,3,4,5,6,7,8 & 9)

Seiji Tomita[1], Oliver Couto[2]

[1]Tokyo Software Company (Inc),Tokyo, Japan
[2]University of Waterloo, Waterloo, Canada
[1]fermat@m15.alpha-net.ne.jp, [2]samson@celebrating-mathematics.com

Abstract: Consider the below mentioned equation: $x^4+y^4+z^4=w*t^n$ (A). Historically Leonard Euler has given parametric solution for equation (A) when w=1 (Ref. no. 9) and degree 'n'=2. Also S. Realis has given parametric solution for equation (A) when 'w' equals 1 and degree 'n' =3. More examples can be found in math literature (Ref. no.6). As is known that solving Diophantine equations for degree greater than four is difficult and the novelty of this paper is that we have done a systematic approach and has provided parametric solutions for degree's 'n' = (2,3,4,5,6,7,8 & 9) for different values of "w". The paper is divided into sections (A to H) for degrees (2 to 9) respectively. $x^4+y^4+z^4=w*t^n - - - - - (A)$

Keywords: Quartic Equation, Diophantine Equations, Pure Math, Number Theory, Sums Of Powers

1. Summary of Background

Whereas in math literature, we find many examples of sums of equal powers, meaning the degree is same on both sides of equation, in this paper we have demonstrated that it is possible to equate parametrically, unequal powers. Meaning the degree on the left hand side of the equation is different from the right hand side of the equations for n = 2, 3, 5, 6, 7, 8 & 9

2. Section (A)

Equation 1

We have the below mentioned equation,

$$x^4+y^4+z^4=w*t^n$$

Let, w=1, degree n=2

$$x^4+y^4+z^4=t^2$$

Let:

$$x = x_0p+1$$
$$y=y_0p$$
$$z=z_0p$$
$$t = t_0p^2+kp+1$$

Where (x_0,y_0,z_0,t_0) are known solutions

$$(x_0p+1)^4+(y_0p)^4+(z_0p)^4=(t_0p^2+kp+1)^2 \qquad (1)$$

We have solution given by Leonard Euler which is given below for,

$$a^4+b^4+c^4=t^2$$

$$\text{a} = 2pq(p^2-q^2),\ \ b=\ (p^2-q^2)(p^2+q^2),$$
$$c=\ 2pq(p^2+q^2), d=\ (p^8+14p^4q^4+q^8)$$

We have, $x^4+y^4+z^4=t^2$

$$let\ (x_0,y_0,z_0,t_0)=(2mn(m^2-n^2),(m^2-n^2)(m^2+n^2),2mn(m^2+n^2),$$
$$(m^8+14m^4n^4+n^8)$$

$Substituting\ in\ equation\ (1) above\ \ we\ get\ the$ $values\ of\ \ (k,p)$ using maple soft

$$k\ =-16m^3n^3\frac{(-3m^2n^4+n^6+3m^4n^2-m^6)}{(m^8+14m^4n^4+n^8)}$$

$$p=$$
$$\frac{-4(n^{10}-m^2n^8+14m^4n^6-14m^6n^4+m^8n^2-m^{10})mn}{n^{16}-8m^2n^{14}+12m^4n^{12}+8m^6n^{10}+230m^8n^8+8m^{10}n^6+12m^{12}n^4-8m^{14}n^2+m^{16}}$$

Substitute k and p to (1), hence we obtain above parametric solution.

$$For\ (x,y,z,t)$$
$$x\ =\ -4m^4n^{12}+\ 128m^6n^{10}+\ 6m^8n^8+\ 128m^{10}n^6-\ 4m^{12}n^4+\ n^{16}+\ m^{16}$$
$$y\ =\ 4(-m^2+\ n^2)(m^2+\ n^2)(n^{10}-\ m^2n^8+\ 14m^4n^6-\ 14m^6n^4+\ m^8n^2-\ m^{10})mn$$
$$z\ =\ -8m^2n^2(m^2+\ n^2)(n^{10}-\ m^2n^8+\ 14m^4n^6-\ 14m^6n^4+\ m^8n^2-\ m^{10})$$
$$t\ =\ (n^{32}+\ 120m^4n^{28}-\ 256m^6n^{26}+\ 2332m^8n^{24}+\ 768m^{10}n^{22}+\ 5960m^{12}n^{20}-\ 512m^{14}n^{18}+\ 48710m^{16}n^{16}-$$
$$512m^{18}n^{14}+\ 5960m^{20}n^{12}+\ 768m^{22}n^{10}+\ 2332m^{24}n^8-\ 256m^{26}n^6+\ 120m^{28}n^4+\ m^{32})$$
$$numerical\ example\ is: (12,15,20)^4=\ (481)^2$$

We next show the parametric solutions of $x^4+y^4+z^4= w*t^2$ for several "w".
In the case of w=2 & 3, parametric solutions and numerical examples are shown.
Identity by S. Realis: w=3, n=2

$$(5a^4+4a^3+9a^2+10a+5)^4+\ (5a^4+10a^3+9a^2+4a+5)^4+\ (5a^4+16a^3+27a^2+16a+5)^4=\ 3\big((a^2+a+1)(25a^6+75a^5+222a^4+319a^3+222a^2+75a+25)\big)^2$$

Search for smallest solutions of $x^4+y^4+z^4= w*t^2$ in the case of w< 20 are as follows.

w	x	y	z	t
1	20	15	12	481
2	2	1	1	3
3	1	1	1	1
9	60	45	36	1443
11	5	5	3	11
17	22	17	6	137
18	2	1	1	1
19	15	15	1	73

Above equation doesn't have a solution in the case of:

w =5,6,7,10,13,14,15(mod 16).

Equation (2)

$$w = 2, \quad \text{degree } n = 2,$$

$$we\ have\ x^4 + y^4 + z^4 = wt^2 \tag{B}$$

We have known solution, (p, q, r, s)=(2,1,1,3)

$$Let\ the\ known\ solution\ be \quad p^4 + q^4 + r^4 = 2s^2$$

$$let\ \ t = sk^2 + gk + h, x = pk + a, y = qk + b, z = rk \tag{C}$$

Substituting equation (B) above in eqn. (A) we get

$$(p\ k + a)^4 + (q\ k + b)^4 + (r\ k)^4 = 2(\ sk^2 + gk + h)^2$$

After some algebra we get,

$$g = \frac{2(ap^3 + bq^3)}{sw}$$

$$h = \left(\frac{1}{2}\right) * \frac{(6a^2p^2 + 6b^2q^2 - g^2w)}{sw}$$

$$k = -\left(\frac{1}{2}\right) * \frac{(a^4 + b^4 - h^2w)}{(2a^3p + 2b^3q - ghw)}$$

Hence equation (B) above has general solution for any ‘w’ & n = 2
We have known solution for w=2, (p, q, r, s)=(2,1,1,3)

After substituting the above we get:

$$g = \frac{8a + b}{3}$$

$$h = \frac{22a^2 + 13b^2 - 8ab}{22}$$

$$k = -\left(\frac{1}{72}\right) * \frac{(239a^3 - 465a^2b + 807ab^2 + 391b^3)}{(7a^2 - 14ab + 34b^2)}$$

We have equations,

$$t = sk^2 + gk + h, \quad x = pk + a, \quad y = qk + b, z = rk$$

After substituting for (g, h, k, p, q, r) we get

$$x = 26a^3 - 78a^2b + 834ab^2 - 782b^3$$

$$y = 239a^3 - 969a^2b + 1815ab^2 - 2057b^3$$

$$z = 239a^3 - 465a^2b + 807ab^2 + 391b^3$$

$$t = 9(6347a^6 - 38082a^5b + 147945a^4b^2 - 337900a^3b^3 + 583689a^2b^4 - 593130ab^5 + 336107b^6)$$

For (a, b) = (1,0) we get $(26,239,239)^4 = 2 * (57123)^2$

Equation 3

For w=3 degree n=2
We have known solution $(1,1,1)^4 = 3(1)^2$

Substituting (p,q,r, s)=(1,1,1,1) in equation (A) we get

$$t = gk + k^2 + h, x = a + k, y = b + k, z = k$$

Hence we arrive at after substitution:

$$x = 23a^4 - 46a^3b + 39a^2b^2 - 16ab^3 + 11b^4$$
$$y = 11a^4 - 16a^3b + 39a^2b^2 - 46ab^3 + 23b^4$$
$$z = 11a^4 - 28a^3b + 57a^2b^2 - 28ab^3 + 11b^4$$
$$t = 321a^8 - 1284a^7b + 2562a^6b^2 - 3192a^5b^3 + 3507a^4b^4 - 3192a^3b^5 + 2562a^2b^6 - 1284ab^7 + 321b^8$$

For (a,b)=(1,1) we get: $(23,11,11)^4 = 3*(321)^4$

Numerical Examples

$$w = 2, n = 2$$

(a, b)= $(1,-3) 1444^4 + 3751^4 + 83^4 = 2 * 10057653^2$
(a, b)= $(1,-2) 362^4 + 959^4 + 47^4 = 2 * 656883^2$

$$w = 3, n = 2$$

(a, b)= $(1,-3) 367^4 + 703^4 + 451^4 = 3 * 318201^2$
(a, b)= $(1,-2) 115^4 + 187^4 + 139^4 = 3 * 24297^2$

$$\textit{we have equation.}\ x^4 + y^4 + z^4 = wt^n$$

Taking, n=3 we get

$$x^4 + y^4 + z^4 = w * t^3$$

We show the parametric solutions of $x^4 + y^4 + z^4 = wt^3$ for w = 1, 2 & 3.
In the case of w =1, 2, and 3, parametric solutions and numerical examples are shown below.
Smallest solutions for:

$$x^4 + y^4 + z^4 = wt^3$$

In the case of "w" for are as follows:

w	x	y	z	T
1	76	72	4	392
2	19	18	1	49
3	228	216	12	1176
5	380	360	20	1960
6	57	54	3	147
7	32	20	12	56
9	106	91	80	297

$$\textit{There are parametric solutions of } x^4 + y^4 + z^4 = w * t^3,$$
$$w = 1, 2, \textit{and } 3.$$

Proof:

we prove the case of w=2, since this is the simplest case.

$$\textit{First, we use the identity } [x^4 + y^4 + (x+y)^4 = 2(x^2 + xy + y^2)^2]$$
$$\textit{To find } (x,y) \textit{ of } (x^2 + xy + y^2) = P^3, \quad \textit{we select 'u' such that } u^3 = 1.$$

Hence (u^3-1) = 0 or (u-1)*(u^2+u+1) = 0

$$\textit{Hence } u^2 = -(u+1), u^3 = 1, u^4 = u \ \& u^5 = -(u+1), u^6 = 1$$
$$\textit{Let}, (x^2 + xy + y^2) = (x - yu)(x - yu^2) = \{(a - bu)(a - bu^2)\}^3.$$
$$\textit{So}, (x - yu) = (a - bu)^3 = (-3a^2b - 3ab^2)u + a^3 - 3ab^2 - b^3$$
$$(x - yu^2) = (a - bu^2)^3$$

$$Hence, \quad we\ obtain\ x = a^3 - 3ab^2 - b^3, y = 3a^2b + 3ab^2, z = x + y = a^3 - b^3 + 3a^2b.$$

3. Section (B)

Equation (4)

Equation (4), $w = 2, n = 3$, substituting above values below we get:

$$Solution\ is, \quad x^4 + y^4 + z^4 = 2t^3$$

$$x = a^3 - 3ab^2 - b^3$$

$$y = 3a^2b + 3ab^2$$

$$z = a^3 - b^3 + 3a^2b$$

$$t = (a^2 + ab + b^2)^2$$

Similarly, we can obtain the parametric solutions of the case w=1 and w= 3.

Equation (5)

We get, w=1, degree n=3

$$x^4 + y^4 + z^4 = w * t^3$$

Since z = (x+y), we get $2(x^2 + xy + y^2)^2 = t^3$

$$For\ w = 1, \quad x^4 + y^4 + z^4 = t^3$$

$$x = -76a^6 + 1080a^4b^2 + 1520a^3b^3 + 60a^2b^4 - 432ab^5 - 76b^6 - 24a^5b$$

$$y = 4a^6 - 1140a^4b^2 - 80a^3b^3 + 1080a^2b^4 + 456ab^5 + 4b^6 - 432a^5b$$

$$z = -72a^6 - 60a^4b^2 + 1440a^3b^3 + 1140a^2b^4 + 24ab^5 - 72b^6 - 456b$$

$$t = 392(a^2 + ab + b^2)^4$$

$$Equation\ (6): \quad w = 3, n = 3 \quad x^4 + y^4 + z^4 = 3t^3$$

Since z = (x + y) we get, $2(x^2 + xy + y^2)^2 = 3t^3$

$$x = -228a^6 + 3240a^4b^2 + 4560a^3b^3 + 180a^2b^4 - 1296ab^5 - 228b^6 - 72a^5b$$

$$y = 12a^6 - 3420a^4b^2 - 240a^3b^3 + 3240a^2b^4 + 1368ab^5 + 12b^6 - 1296a^5b$$

$$z = -216a^6 - 180a^4b^2 + 4320a^3b^3 + 3420a^2b^4 + 72ab^5 - 216b^6 - 1368a^5b$$

$$t = 1176(a^2 + ab + b^2)^4$$

Numerical Examples:

$$w = 1, \ degree\ n = 3$$

$$(a, b)$$

(a,b)=(1, 0) $\quad 76^4 + 4^4 + 72^4 = 392^3$

$$for\ (2, 1) \quad 23108^4 + 27212^4 + 4104^4 = 941192^3$$

For, $\quad w = 2, n = 3$

(a, b) for (2, 1) $\quad 1^4 + 18^4 + 19^4 = 2 * 49^3$

w = 3, n=3

(a, b)= (1, 0) , $228^4 + 12^4 + 216^4 = 3 * 1176^3$

(a, b)= $for\ (2, 1) \quad 69324^4 + 81636^4 + 12312^4 = 3 * 2823576^3$

4. Section (C)

$$x^4 + y^4 + z^4 = w * t^n$$

degree n=4

$x^4+y^4+z^4=w*t^4$.

For degree ‘n’

Consideration of $x^4+y^4+z^4=wt^4$ (mod 16). In the case of (n mod 16)=4,5,6,7,8,9,10,11,12,13,14,15, above equation has no solutions.

Consideration of $x^4+y^4+z^4=w*t^4$ (mod 25)

In the case of (n mod 25) = 4, 5, 9, 10, 14, 15, 19, 20, 24 above equation has no solutions.

For w=2

There are infinitely many solutions.

Using identity $x^4 + y^4 + (x + y)^4 = 2(x^2 + xy + y^2)^2$, we can get a parametric solution.

One of the solution $for\ x^2 + xy + y^2$=1 is (x, y) = (1, 0), so we obtain a following parameter solution.

$$(-1+k^2)^4+ (k^2+2k)^4 + (1+2k)^4 = 2(1+k+k^2)^4$$

Numerical solutions for above for w=2 are given below:

k	x	y	z	t
2	8	3	5	7
3	15	8	7	13
5	35	24	11	31
6	48	35	13	43
7	21	16	5	19
8	80	63	17	73
9	99	80	19	91

2. For, w=18

There are infinitely many solutions.

In the same way as w=2, we can get a parameter solution.

$x^2+xy+y^2=3$ is (x, y)=(1,1)

$$(-1+2k+2k^2)^4 + (-2-2k+k^2)^4+(1+4k+k^2)^4 = 18(1+k+k^2)^4$$

3. For, w=98

There are infinitely many solutions.

In the same way as n=2, we can get a parameter solution.

$x^2+xy+y^2=7$ is (x, y)=(2,1)

$$(-3-2k+2k^2)^4+ (-1+4k+3k^2)^4+(2+6k+k^2)^4 = 98(1+k+k^2)^4$$

5. Section (D)

w=1 & degree n=5

Equation (7)

$$x^4 + y^4 + z^4 = t^5$$

$$x = 5580a^8b^2 + 35760a^7b^3 - 31248a^5b^5 + 36540a^6b^4 - 20880a^3b^7 - 62580a^4b^6$$
$$+5580a^2b^8 - 1740a^9b + 2980b^9a - 298a^{10} + 174b^{10}$$
$$y = -13410a^8b^2 - 20880a^7b^3 + 75096a^5b^5 + 26040a^6b^4 - 14880a^3b^7 + 36540a^4b^6$$

$$-13410a^2b^8 - 1240a^9b - 1740b^9a + 174a^{10} + 124b^{10}$$

$$z = -7830a^8b^2 + 14880a^7b^3 + 43848a^5b^5 + 62580a^6b^4 - 35760a^3b^7 - 26040a^4b^6$$

$$-7830a^2b^8 - 2980a^9b + 1240b^9a - 124a^{10} + 298b^{10}$$

$$w = 98(a^2 + ab + b^2)^4$$

Numerical solution is;
(a, b) = (1, 0) $298^4 + 174^4 + 124^4 = 98^5$

$$(a, b) = (2, 1) \quad 5008486^4 + 2084068^4 + 2924418^4 = 235298^5$$

6. Section (E)

$$x^4 + y^4 + z^4 = wt^n$$

w=2 & degree n=6

Equation (8)

$$X^4 + y^4 + z^4 = 2t^6$$

We have the known identity given below,

$$(a^3 + 3a^2b - b^3)^4 + (-a^3 + 3ab^2 + b^3)^4 + (3a^2b + 3ab^2)^4 = 2 * (a^2 + ab + b^2)^6 \qquad (2)$$

We have known solution $(1,19,18)^4$= 2∗ $(7)^6$

Since $(a^2 + ab + b^2) = 7$ *when* $(a, b) = (2,1)$
Hence we *let* $a = 2 + m$ *and* $b = (1 + km)$ in equation (1) above
We get $m = -(4k + 5)/(k^2 + k + 1)$

After substituting this in equation (2) we get the below mentioned parametrization:

$$x = m^6 - 108m^5 - 285m^4 - 20m^3 + 270m^2 + 114m + 1$$

$$y = 19m^6 + 6m^5 - 270m^4 - 380m^3 - 15m^2 + 108m + 19$$

$$z = 18m^6 + 114m^5 + 15m^4 - 360m^3 - 285m^2 - 6m + 1$$

$$t = 7(m^4 + 2m^3 + 3m^2 + 2m + 1)$$

Hence we get the new Identity:

$$(m^6 - 108m^5 - 285m^4 - 20m^3 + 270m^2 + 114m + 1)^4 +$$

$$(19m^6 + 6m^5 - 270m^4 - 380m^3 - 15m^2 + 108m + 19)^4 +$$

$$(18m^6 + 114m^5 + 15m^4 - 360m^3 - 285m^2 - 6m + 18)^4$$

$$= 2[\,7(m^4 + 2m^3 + 3m^2 + 2m + 1)]^6 \qquad (B)$$

For m=2 in above eqn. we get: $(1026, 6803, 5777)^4 = 2 * (343)^6$

$$x^4 + y^4 + z^4 = wt^7$$

w =1 & degree n=7

7. Section (F)

Equation (9)

$$For\ w = 1, \quad x^4 + y^4 + z^4 = t^7$$

$$x = -272832a^{13}b - 272832ab^{13} + 13888b^{14} - 33376a^{14} - 100228128a^8b^6$$

$$-27803776a^9b^5 + 12148864a^{11}b^3 + 1263808a^{12}b^2 + 41705664a^6b^8$$

$$-66882816a^7b^7 + 19507488a^{10}b^4 + 66818752a^5b^9 - 5055232a^3b^{11} + 19507488a^4b^{10} - 3037216a^2b^{12}$$

$$y = -194432a^{13}b - 194432ab^{13} - 33376b^{14} + 19488a^{14} + 58522464a^8b^6$$
$$+66818752a^9b^5 - 7093632a^{11}b^3 - 3037216a^{12}b^2 - 100228128a^6b^8$$
$$-47663616a^7b^7 + 13901888a^{10}b^4 - 39014976a^5b^9 + 12148864a^3b^{11}$$
$$+13901888a^4b^{10} + 1773408a^2b^{12}$$
$$z = -467264a^{13}b - 467264ab^{13} - 19488b^{14} - 13888a^{14} - 41705664a^8b^6$$
$$+39014976a^9b^5 + 5055232a^{11}b^3 - 1773408a^{12}b^2 - 58522464a^6b^8$$
$$-114546432a^7b^7 + 33409376a^{10}b^4 + 27803776a^5b^9 + 7093632a^3b^{11}$$
$$+33409376a^4b^{10} - 1263808a^{2b^{12}}$$
$$t = 392 * (a^2 + ab + b^2)^4$$

Numerical solution for above is:

(a,b) =$(1,0)\,33376^4 + 19488^4 + 13888^4 = 392^7$

(a,b)=$(2,1)\quad 3862651968^4 + 21749064736^4 + 25611716704^4 = (941192)^7$

8. Section (G)

Degree n = 8

$$x^4 + y^4 + z^4 = wt^8$$

For w=2, n=8

Equation (10)

$$x^4 + y^4 + z^4 = 2\,t^8$$
$$Let\ X = a, \quad Y = b, \quad Z = a + b$$
$$We\ have\ the\ identity,$$
$$X^4 + Y^4 + Z^4 = 2 * (a^2 + ab + b^2)^2 \tag{1}$$
$$Take \quad a = m^4 - 6m^2n^2 - 4mn^3$$
$$b = -(-4m^3n + n^4 - 6m^2n^2)$$

then the right hand side of above equation (1)becomes after substitution of

$$(a,b) \quad a^2 + ab + b^2 = (m^2 + mn + n^2)^4.$$
$$We\ obtain\ a\ parametric\ solution\ as\ follows.$$
$$X = m^4 - 6m^2n^2 - 4mn^3$$
$$Y = 4m^3n - n^4 + 6m^2n^2$$
$$Z = m^4 - 4mn^3 + 4m^3n - n^4$$
$$t = m^2 + mn + n^2$$

For (m,n) = (2,1) we get numerical solution as (x,y,z,w) = (16,55,39,7)

9. Section (H)

Degree n=9
For w=2, n=9

Equation (11)

A parametric solution of

$$X^4 + Y^4 + Z^4 = 2t^9.$$

Let X=a, Y=b, Z=a+b

Hence

$$X^4 + Y^4 + Z^4 = 2(a^2 + ab + b^2)^2$$

$$Hence \qquad t^9 = (a^2 + ab + b^2)^2$$

$$Set \quad a = m^9 - 36m^7n^2 + 126m^4n^5 - 9mn^8 - 84m^6n^3 + 84m^3n^6 - n^9$$

$$\text{and,} \quad b = 9m^8n - 126m^5n^4 + 36m^2n^7 + 36m^7n^2 - 126m^4n^5 + 9mn^8,$$

$$Then \qquad a^2 + ab + b^2 = (m^2 + mn + n^2)^9.$$

We obtain a parametric solution as follows.

$$X = m^9 - 36m^7n^2 + 126m^4n^5 - 9mn^8 - 84m^6n^3 + 84m^3n^6 - n^9$$

$$Y = 9m^8n - 126m^5n^4 + 36m^2n^7 + 36m^7n^2 - 126m^4n^5 + 9mn^8$$

$$Z = m^9 - 84m^6n^3 + 84m^3n^6 - n^9 + 9m^8n - 126m^5n^4 + 36m^2n^7$$

$$W = (m^4 + 2m^3n + 3m^2n + 2n^3m + n^4)$$

Numerical solution for (m, n) = (1,1) we get: $(x, y, z)^4 = (81,81,162)^4 = 2(9)^9$
w=1, n=9

Equation (12)

$$A\ parametric\ solution\ of \quad X^4 + Y^4 + Z^4 = t^9$$

Let X=p, Y=q, Z=p+q
Hence

$$X^4 + Y^4 + Z^4 = 2(p^2 + pq + q^2)^2 = t^9$$

$$X = -4163436a^{16}b^2 + 233152416a^{13}b^5 - 1190742696a^{10}b^8 + 23108a^{18} + 23108b^{18}$$
$$-35163072a^5b^{13} - 83268720a^4b^{14} + 865994688a^7b^{11} - 18856128a^{15}b^3 + 428976912a^{12}b^6$$
$$-1123510960a^9b^9 + 179582832a^8b^{10} + 12558240a^{14}b^4 - 130605696a^{11}b^7 + 627912a^2b^{16}$$
$$+489816ab^{17} + 428976912a^6b^{12} - 18856128a^3b^{15} - 73872a^{17}b$$

$$Y = 36(-2b^2 + 2ab + 3a^2)(b^2 + 6ab + 2a^2)(-3b^2 - 4ab + a^2)$$
$$* (b^6 - 108b^5a - 285b^4a^2 - 20b^3a^3 + 270b^2a^4 + 114ba^5 + a^6) *$$
$$(19b^6 + 6b^5a - 270b^4a^2 - 380b^3a^3 - 15b^2a^4 + 108ba^5 + 19a^6)$$

Then we arrive at:

$$p^2 + pq + q^2 = (2)^4 * (7)^9 * (a^4 + 2a^3b + 3a^2b + 2ab^3 + b^4)^9$$

We obtain a parametric solution as follows:

$$Z = -627912a^{16}b^2 + 35163072a^{13}b^5 - 179582832a^{10}b^8 + 27212a^{18} + 27212 * b^{18}$$
$$+197989344a^5b^{13} - 12558240a^4b^{14} + 130605696a^7b^{11} - 22204992a^{15}b^3 + 505163568a^{12}b^6$$
$$-1323047440a^9b^9 - 1011159864a^8b^{10} - 70710480a^{14}b^4 + 735388992a^{11}b^7 - 3535524a^2b^{16}$$
$$+73872ab^{17} + 505163568a^6b^{12} - 22204992a^3b^{15} + 415944a^{17}b.$$

$$t = 2 * 7^2 * (a^4 + 2a^3b + 3a^2b + 2ab^3 + b^4)^2$$

Numerical solution is: For (a, b) =(1,1)

$$[x, y, z]^4 = [-454834764, -80779032, -535613796]^4 = [7938]^9$$

10. Conclusions

This paper has analyzed the equation. $\mathbf{x^4 + y^4 + z^4 = w * t^n}$, for n = (2,3,4,5,6,7,8,9). In the near future attempt can be made by others to find solution for degree n>9 and for different values of integer 'w'.

REFERENCES

[1] Seiji Tomita, Self web page, Computation number theory: http://www.maroon.dti.ne.jp/fermat.

[2] Published math paper, Oliver Couto & Seiji Tomita, Generalized parametric solution to multiple sums of powers, Universal Journal of applied mathematics, Dec 2016, Volume 3(5), pages 102-111,http://www.hrpub.org.

[3] Published math paper, Oliver Couto & Seiji Tomita, Solution of Polynomial Equation of any degree 'n' with Special Emphasis for n= (2, 3, 4, 5 & 6), Journal "SOP transactions on Applied mathematics" ISSN(Print): 2373-8472, July 2015.http://www.scipublish.com/journals/am.

[4] Published Math paper, Oliver Couto, Taxicab Equations for power (2, 3, 4 & 5), Journal, International Math Forum, Hikari ltd., Vol.9, Jan 2014 no.12, pages 561-577. http;//www.m-hikari.com/imf.html.

[5] Oliver Couto, Self, web page on Mathematics, http://www.celebrating-mathematics.com.

[6] SeijiTomita,Fourthpowerpolynomialequation, Computation number theory- web-page,http://www.maroon.dti.ne.jp/fermat/diop122e.html

(7)SeijiTomita,sixthpowerpolynomialequation,Computation Number theory-Web-page,http://www.maroon.dti.ne.jp/fermat/diop160e.html.

8)Ramanujan lost notebook, Narosa publishing house, NewDelhi, India.

9)Euler Leonhard, Opera Omnia, 1984.

10)Tito Piezas-Online collection of algebraic identities http://sites.google.com/site/tpiezas.

11)Ajai Choudhry, Symmetrical Diophantine equations, Journal of mathematics, Rocky mountain journal vol. 34,no.4,winter(2004), pg. 1261-1298.

12)Jaroslaw Wroblewski, Tables of Numerical solutions for degree three, four, six, seven &nine, website, www.math.uni.wroc.pl/~jwr/eslp.

13)L. E. Dickson, history of the theory of numbers, Vol.II, (Diophantine analysis), AMS Chelsea publication, year 2000.
